General Relativity Consistently Unified with Quantum Theory

Giving **Quantum Gravity** in Operator Notation **+ Dark Energy**

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Abstract:

This unification is achieved by group theory. The group SU(2,2) is identified to be the fully quantized covering group of (an extended form of) GR.

(1) Its linear generators yield quantum theory,
(2) Its non-linear Casimir operators create curvature,
(3) Irreducibility provides background independence.

Dark energy results as a byproduct of the commutation relations of the (non-linear) space-time operators. By the number 3 of Casimir operators in an SU(2,2), the equations of motion in GR are fixed to have three components (3-dimensionality of motion).

A maximal set of commuting generators defining the " quantization axis" of GR is given by the triplet L_3 (spin), Q_3 (CMS-space), P_0 (energy).

100 years ago, 5 discoveries revolutionized fundamental physics:

- ✓ Planck — discovered: **Our world is quantized**.
- ✓ Einstein — found **General Relativity** in his differential-geometric representation.
- ✓ Discovery of — **half-integral spins:** Our world is not orthogonal but unitary ("covering group").
- ✓ Dirac — postulated **antiparticles** by introducing his γ-matrices.
- ✓ DeSitter's — **group contraction** yielded the Poincaré group as a limit of an SO(2,3) or SO(1,4).

With half a century delayed, then,

- ✓ Gell-Mann's — **quarks**, finally, demonstrated: elementary particles are **composed** structures.

These are the ingredients for a *comprehensive*, *consistent* quantum gravity. It is the first time that a direct, **non-perturbative** unification of Planck's theory with Einstein's theory is established!

Why hasn't it been established earlier? The open questions since all those times had been:

- the **quantization of space-time** (i.e., that „missing link“ Planck/Einstein), allowing
- the **quantization of General Relativity.**

The unification of Planck with Einstein had failed due to **one fundamental problem:**

QM linear (superposition principle),
GR non-linear (space-time is bent).

Contrary to public opinion, however, both properties **do *not* contradict** each other!

But, first of all, let us start with some no-go theorems. First: A limit *inversion* never is unique. GR, as a *continuous* model using *limits* (cf. its Coulomb singularity, e.g.), thus, can*not* uniquely be reduced to an *atomistic* model. Hence, not GR has to be quantized – but, inversely, quantum theory must be shown to yield GR as some limiting substructure:

GR is secondary, quantum theory is primary !

Second: on a bent surface, the parallelogram of vectors, usually, will not close up. Thus,

> **The curvature of space-time is stating that the operators X_μ**
> the space-time components are assumed to be the eigenvalues of
> **are not commuting with each other!**

(Otherwise, the parallelogram would close up.)

A "canonical quantization", however, is based on *commuting space-time* operators. Thus,

> A **canonical** quantization never will have a chance to reproduce GR
> (**or:** a metric artificially introduced *from outside*
> will prevent it from being "background-independent").

Hence,

> Neither **Lagrangian** models, as used in the "standard model",
> nor **path integrals**, as used in string/brane models, ever will succeed!

And geometrical playgrounds like Loop Quantum Gravity or Triangulation Models dealing with 4 <u>commeasurable</u> space-time components are ignoring the undisputed fact that

non-commuting operators are ***not* measurable** simultaneously:

For, they are applying "geometrical eigenvalues" of non-commuting operator components X_μ – always in a change.

It took 100 years, but now we have got the **solution** I am deriving immediately**:**

Quantum Gravity

is nothing else than the

physics of the group

SU(2,2) !

– such as the conventional quantum *field theories* are the physics of the group SL(2,c), which, essentially, is the Lorentz group SO(1,3) – or its extension to a Poincaré group, respectively.

Proof (that linear QM and non-linear GR do not contradict each other) by a **counter-example:**

As a Lie algebra, Dirac's γ-matrices are generating a **group U(2,2)** with 16 generators G_a here renamed as

L_0	M_0	P'_0	Q'_0
L_1	M_1	P'_1	Q'_1
L_2	M_2	P'_2	Q'_2
L_3	M_3	P'_3	Q'_3

L_0 is the linear, the „**1st-order Casimir operator**"; $[L_0,G_a]_- = 0$. *(Casimir operators of all orders are commuting with all generators; they are the constants with respect to all transformations.)* Without L_0, we are left with a 4-dimensional

SU(2,2) = covering group of a 2+4 = **<u>6-dimensional</u>** "**conformal group**" **SO(2,4)**.

The transition SU(2,2) <-> SO(2,4) proceeds by resorting + reflecting + renaming**:**

0	$-P'_0$	$-M_0$	$-P'_3$	$-P'_2$	$-P'_1$
$+P'_0$	0	$+Q'_0$	$-M_3$	$-M_2$	$-M_1$
$+M_0$	$-Q'_0$	0	$-Q'_3$	$-Q'_2$	$-Q'_1$
$+P'_3$	$+M_3$	$+Q'_3$	0	$+L_1$	$-L_2$
$+P'_2$	$+M_2$	$+Q'_2$	$-L_1$	0	$+L_3$
$+P'_1$	$+M_1$	$+Q'_1$	$+L_2$	$-L_3$	0

=

0	$+L_{65}$	$+L_{64}$	$+L_{63}$	$+L_{62}$	$+L_{61}$
$+L_{56}$	0	$+L_{54}$	$+L_{53}$	$+L_{52}$	$+L_{51}$
$+L_{46}$	$+L_{45}$	0	$+L_{43}$	$+L_{42}$	$+L_{41}$
$+L_{36}$	$+L_{35}$	$+L_{34}$	0	$+L_{32}$	$+L_{31}$
$+L_{26}$	$+L_{25}$	$+L_{24}$	$+L_{23}$	0	$+L_{21}$
$+L_{16}$	$+L_{15}$	$+L_{14}$	$+L_{13}$	$+L_{12}$	0

Yellow: The 6 Lorentz-generators (L_i,M_i).

Green: According to deSitter's group contraction, the P'_μ may be interpreted as 4-momentum.

But what, now, are the $\mathbf{M_0}$ and $\mathbf{Q'_\mu}$? A first trial, to identify Q'_μ = space-time X_μ, failed because, as a generator of a linear group, Q'_μ is an additive, conserved quantity – while X_μ is neither additive nor conserved.

But let us consider the **2^{nd}-order Casimir-Operator:**

$$C^{(2)}_{SU(2,2)} \equiv +\left(\underline{P_0'^2 - \vec{P}'^2 - M_0^2}\right) - \left(Q_0'^2 - \vec{Q}'^2\right) - \vec{M}^2 + \vec{L}^2 .$$

Underlined, it contains a KG term. Hence, we may interpret

$C^{(2)}$ = generalized Klein-Gordon operator
with M_0 = heavy mass.

The terms outside that KG-kernel are corresponding to Einstein's **cosmological constant** λ. (Its discussion would need a separate lecture.) λ, thus, by no means is constant!

{ Now, for an irreducible representation, **Casimir = constans**.
On the other hand, $C^{(n)}$ is a polynomial of n^{th} order in the generators. } ⇒

!!! In its macroscopic limit *(Hermit. operators → r-numbers)* $C^{(n)}$ will yield an (n–1)-dimensional, ***non-linear***, ***bent*** **hyperspace** (GR) within the n-dimensional space of a ***linear*** **group** (QM) **!** !!!

Both aspects – the linear one of generators and the non-linear of the (higher) Casimirs – are present *simultaneously*!
This solves the above 100 years old **compatibility problem Planck/Einstein**! (q.e.d.).

Physics, of course, is exclusively proceeding *within* that hyperspace**:** There is no group transformation leading outside its irreducibility boundaries into its **embedding** neighbourhood! This exactly is Einstein's **background independence** of GR! Thus,

Irreducibility is slicing
parameter space
into bent universes
orthogonal to each other.

SU(2,2) may be considered as an "extended Poincaré group" with ***3 Casimirs***. We immediately are going to identify space-time X_μ as some non-linear functions $\propto L_{\mu 4}$ of generators – just wait a minute. Then, however, the **3** Casimirs set = const. allow a resolution with respect to, say, $\vec{X}$, giving

- location = f(time, ...). This, however, is an **equation of motion.** As it is derived from
- 3 Casimirs, it will give rise to exactly **3 components** *(and not 4 or 2!)***:**

$$\vec{X} = \vec{f}\left(X_0, D, P_\mu, \vec{M}, \vec{L}; C^{(2)}_{SU(2,2)}, C^{(3)}_{SU(2,2)}, C^{(4)}_{SU(2,2)}\right) .$$

This **3-dimensionality of motion**, of course, has nothing to do with the 3-dimensionality of "space" $\propto L_{i4}$ with $_i = {}_{1,2,3}$! *(The 3-dimensionality of space is connected with the number 3 of generators of the spin subgroup SU(2) ≅ SO(**3**) – and not from SU(2,2)-Casimirs.)*

Like the generators themselves, **non-linear space-time**, as some function of these generators, **is fully quantized**, too! Only in their *macroscopic limits*, then, all those operators are commuting with each other, and their discrete eigenvalues are smearing out to a continuum. This continuum permitted Einstein to apply his *diff.-geometrical* methods.
From the Heisenberg view of QM, we thus obtained**:**

SU(2,2)

is the covering group of

General Relativity

← fully quantized,

← in its macroscopic limit.

Quantized space-time X_μ:

By $c = 1 = \hbar$, beyond Lorentz symmetry still 2 measuring units are left open. Let us call them ℓ, q. If not set equal to 1, $\hbar$ would have become the 3rd parameter unit for the following 3 re-normalizations:

$$\begin{aligned} P_\mu &\equiv \tfrac{1}{\ell} P'_\mu, \\ Q_\mu &\equiv \ell q Q'_\mu, \\ D &\equiv q M_0. \end{aligned}$$

(Observe that this ℓ is not the Planck length!)

By group contraction, the coefficients on the right-hand side, as shown below, → 0. (In physics, this means: they are „extremely small“.) In this *limit*, all those operators would commute with each other (see right-hand sides "→ 0"):

$$\begin{aligned}
[P_{\mu'}, P_{\mu''}]_- &= \tfrac{1}{\ell^2}\left(+i\varepsilon_{\mu'\mu''\mu} L_\mu + i\delta_{\mu'0} M_{\mu''} - i\delta_{\mu''0} M_{\mu'}\right) \xrightarrow{1/\ell^2 \to 0} 0, \\
[Q_{\mu'}, Q_{\mu''}]_- &= (\ell q)^2 \left(-i\varepsilon_{\mu'\mu''\mu} L_\mu - i\delta_{\mu'0} M_{\mu''} + i\delta_{\mu''0} M_{\mu'}\right) \xrightarrow{(\ell q)^2 \to 0} 0, \\
[P_{\mu'}, Q_{\mu''}]_- &= -i(-1)^{\delta_{\mu'0}} \delta_{\mu'\mu''} D \left(\xrightarrow{\hbar \to 0} 0\right), \\
[D, P_\mu]_- &= +i \tfrac{1}{\ell^2} Q_\mu \xrightarrow{1/\ell^2 \to 0} 0, \\
[D, Q_\mu]_- &= +i \ell^2 q^2 P_\mu \xrightarrow{\ell^2 q^2 \to 0} 0.
\end{aligned}$$

The central [P,Q]-commutator is resembling the [P,X] leading to Heisenberg's uncertainty relation; only the mass term $D \propto M_0$ would have to be dropped. Somehow, hence, we should divide by that D and combine it with the Q_μ **:** Therefore, define the space-time operator X_μ as their quotient:

$$X_\mu = Q_\mu / D,$$

Hermitean form: $2\, D X_\mu D = [D, Q_\mu]_+$.

Then, the additive $\mathbf{Q_\mu}$ becomes „**CMS-space-time**“ ($\propto$ heavy mass D times X_μ).

While the non-commutativity of the $[P,Q]_-$ uncertainty commutator long since is proved *directly* by experiment, for the non-commutativity of the remaining commutators evidence actually only is *indirect* by Einstein's curvature in GR – I pointed to that in the beginning.

Now, the uncertainty commutator with X replacing Q will provide an additional term, whose contraction (far on right-hand side), finally, yields Heisenberg's form (with $\hat{X}$ from $Q_\mu \equiv \hat{X}_\mu D = D \hat{X}_\mu^{\,+}$):

$$[P_{\mu'}, X_{\mu''}]_- = -i\left((-1)^{\delta_{\mu'0}} \delta_{\mu'\mu''} - \underline{\tfrac{1}{2\ell^2}\left(\hat{X}_{\mu''}\hat{X}_{\mu'} + \hat{X}^+_{\mu'}\hat{X}^+_{\mu''}\right)}\right) \xrightarrow{(1/\ell^2) \to 0} -i(-1)^{\delta_{\mu'0}} \delta_{\mu'\mu''}.$$

Our ℓ, hence, is some length unit >>1. In the *close* range, that

additional term < error bars of its measuring uncertainty.

Thus, by experiment, it will be observable only from some *minimal threshold distance* on and further out. This effect, however, is well known**:**

ℓ will **be measurable** by feeding in the data of **Dark Energy.**

Dark Energy,
hence, unveils as some
quantum effect
on cosmic scale!

Quantizing GR

A maximal set of commuting, *additive*, compact SU(2,2) quantum numbers is provided by**:**

$L_{12} = L_3$ = spin component,
$L_{34} = Q'_3$ = its respective CMS space component,
$L_{56} = P'_0$ = energy,

giving

$$\psi = \psi(L_3, Q'_3, P'_0, ...) .$$

The **eigenvalues** of those 3 spin-like quantum numbers are multiples of ½ , each. Thus, according to sign combinations, there is a fundamental set of 8 "basic quanta"**:**

$$\psi(L_3, Q'_3, P'_0) = \psi\left(\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\right) .$$

Higher U(2,2)-representations are available in the mathematical literature.

In any case, like in good old QM**:**

Quantization axes (pointing to the *unitary "2-dimensionality of spin"* *replacing the "3-dimensionality of space"*) are inevitable also in **General** Relativity**.**

This will cancel the infrared Coulomb singularity, e.g., as I could show (with some more time). Etc., etc.

Result:.

SU(2,2) is an extremely simple model giving rise to complete QG in a mathematically closed, **non-perturbative** representation. All you need is a little bit group theory beyond spin and the Lorentz group. It is by far less sophisticated and unlike more powerfull than the *successless* complexity of the string/brane models – those flops of the century.

Correspondingly, the **"standard model"** (of particles) **is a collection of parameters** shrinked

- **to fit the *variation* principle** (i.e., path integrals, Lagrangians, ...) of classical point mechanics,
- trimming it by a "canonical quantization"
- plus some ad-hoc doctrines (i.e., parameter providers like gluons, Higgs, …)

whose relevance for physics is as questionable as that of strings and branes. People are trying to persuade us that quantum theory is delimited to Heisenberg's uncertainty commutator (which is providing a substitution $p_\mu \to \pm i\partial_\mu$)**.** However, QG and its GUT-extension [2] [3] already proceeded far beyond that

centennial dead end of fundamental physics!

Conclusion:.

The "**standard model**" (of particles) – and with it the string/brane models – are restricting themselves to

- **canonical conjugation**, i.e., to the *variation principle* with Lagrangians, path integrals, **…** ,
- with just **ONE time** direction
- and with **commuting space-time** components**.**

Now, where we are in possession of the solution to quantum gravity, we may retrospectively formulate**:** People were trying to solve the challenges of the 21st century by hedging the conservative concepts of good old point mechanics of the 18th century, ignoring that time had gone on in the meantime. They just enlarged the number of *classical parameters* (points → "strings" → "branes") – but without **rethinking** their deeply classical ***physical conceptions*** behind**:** They cemented quantum theory just to be the outcome of Fourier transformations and of applying the *classical variation* principle.

Now, **quantum gravity**, instead, revealed simply to be *nothing else than* the

physics of the group SU(2,2),

- ✓ with **NON-commuting space-time** operators, which are giving rise to dark energy,
- ✓ with **GR as the macroscopic limit** of its locally isomorphic conformal group SO(2,4) treated already in the dawn of the "golden" **19**20-s
- ✓ with **TWO time** and four space directions!

We need not take recourse any more to approximations like LQG or triangulation models. Now, we can calculate everything directly, exactly. **Quantum gravity now is available for everybody.**

Just apply it!

Quantum Gravity

is the

physics of the group

SU(2,2) !

*My recommendation only can be***:** *Dare the leap from the losing horse of strings and branes over to the winning horse of QG and GUT !*

[1] C. Birkholz, Verhandl. DPG (VI) **47,1**/GR 10.1 (2012).
[2] C. Birkholz, Verhandl. DPG (VI) **46,3**/T 25.1 (2011).
[3] C. Birkholz, "Weltbild *nach Vereinheitlichung aller Kräfte der Natur* im 3. Jahrtausend", ISBN 978-3-00-030847-5 (2010).

Free copies on request. *(My postcard address is in the conference catalogue.)*