

Abstract:

The recent breakthrough resulting in a non-perturbative unification of Einstein's relativity with Planck's quantum theory has posed onward questions:

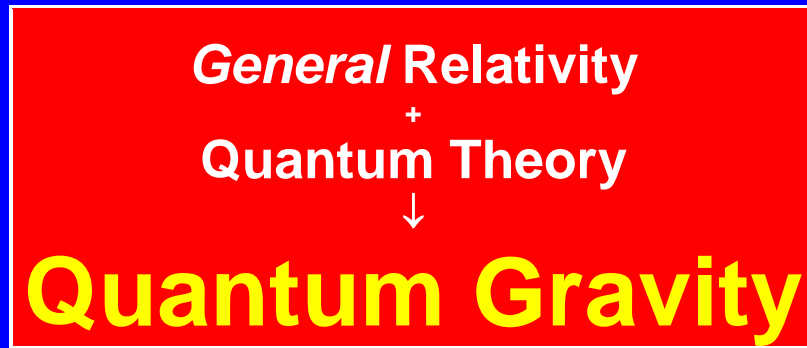
- Why is (bent) space-time (1+3)-dimensional?
- Why is it an $SU(2,2)$ which is the covering group of fully quantized General Relativity?

Only a handful of more or less evident postulates is needed in order to deduce that fundamental physics must be based on an atomistic model and that the number of degrees of freedom should be some small power of 8 in terms of a degeneracy expansion. Irreducibility, then, is slicing our world into separate bent universes orthogonal to each other.

After deriving Einstein's "World Formula" from first principles, its evaluation provides a consistent Quantum Gravity in fully quantized bent space-time on its first degeneracy level – as a singlet representation with respect to the Grand Unification (GUT). Its second degeneracy level adds "internal" forces as vector representations of the GUT. A particle (viewed from outside) and our universe (viewed from inside) are subject to identical equations.

Finally, the mechanism how a particle is condensing out of Dark Matter is derived.

Einstein's dream of a **"World Formula"** as the quintessence of combining his



found its first **consistent** and **analytic** (*i.e., non-perturbative*) formulation in 2010 [2], when GR, including space-time, was fully quantized **without violating "background independence"**.

It was formulated neither in the working horse of particle theoreticians, *i.e., variation principle, path integrals, Lagrangians*, nor in Einstein's differential geometry used by the gravitationalists, but in terms of a 3rd category: in **group theory** – the best-hated mathematics for *both* types of physicists. Schrödinger even rebaptized it **"group pestilence"**. It had to be expected that only some hitherto neglected field of mathematics could overcome the routine-blindedness of particle theoreticians and Einstein's disciples!

My first ansatz produced the mathematical group "SU(2,2)" as the "covering group" of fully quantized GR. Its number 3 of constants, the "Casimir operators" characterizing that group, had been shown to give rise to the

3-dimensionality of motion:

$$\vec{X} = \vec{f}(X_0, \dots),$$

within Einstein's 4-dimensional **bent space-time**. Thus, **motion is not caused by** the minimalization procedure of the variation principle falling from heaven, but by the mere constraints of **Casimir operators!**

Casimirs, in general, are the source of all a group-theoretical model *of the types used here* can tell us about the **laws of physics**. Hence, Einstein's "**World Formula**" must read:

$$\text{Casimir} = \text{const.}$$

valid for all related Casimirs existing – whatever the *group-theoretical* model might be. Einstein himself never succeeded in finding this simple formula *because he didn't care about group theory!*

Since Einstein's times, however, the experimental situation has changed dramatically. A veritable "zoo" of elementary particles has entered the tables showing up diverse "**internal forces**" in addition to gravity. In this regard, Einstein's old search for a QG better should be actualized to an onward search for the "Grand Unification Theory" (GUT) combining



A preliminary formulation of such a GUT had already been given within the context of QG [2]. Here, I am presenting their first *comprehensive* formulation. Both novel formulations – that of QG [2] [3] as well as that of a GUT [2] – **do not distinguish between our universe and an elementary particle**. (This is a property of QG.) Both descriptions are quoting exactly **the same fundamental equations!** Their difference just lies in their point of observation:

A **particle** is observed from outside,
our universe from inside !

Now, what is the model? It is not the “standard model”. Which are the “**First Principles**”? Let us start with the least specific ones.

Principle #1:

Theoretical **physics** is the *mapping* of
(parts of) **nature** into **mathematics**.

(Only the string/brane protagonists are that “progressive” that they even cancelled “**nature**” as their point of reference !)

Now, we humans are able to count natural numbers. But we are not able to continue that job up to infinity – somewhere, we shall have to stop

Principle #2:

In physics, everything must remain **finite**,
in physics, there are **no singularities**.

However, mathematical **limits do** imply **infinities!** Hence,

Mathematical limits are unphysical.

At most, **they are denoting approximations**.

If we want to “**understand**” **physics**, we should try to design some model excluding features which *already proved* to be “unphysical”. Thus, **continous models** rely on *limiting* procedures on natural numbers. Hence,

Only **atomistic models**
can be strictly “physical”

from the beginning !

Let us, hence, start with an *atomistic modelling* and designate its “atoms” as “**quanta**”.

Principle #3:

All those **quanta** are assumed to be **independent** of each other and, on principle, **individually distinguishable** from each other.

Remark to the mathematicians: The mathematics applicable to such a set of **quanta** q_n ($n = 1, \dots, N$) is **set theory**. Its properties permit a representation of the “union” of separate quanta as their multiplication.

Principle #4:

Physics is unthinkable *with* **out** its **error bars**.

Theoreticians, usually, are ignoring this problem! This central trait, however, is introducing a concept of **probability** into physics! *Mathematically, this means the addition of some field of numbers to our quanta. And, probability must be normalizable. This needs an operation of division. Now, the mathematicians are teaching us that such a field of numbers which includes the operation of division is confined to have 8 dimensions maximally. (Its “numbers”, then, are called “octonions”. Confer the “imaginary plane”, which is an example of some 2-dimensional field of numbers spanned by “1” and “i”.) Our result from mathematics reads:*

Principle #5:

**Our world looks
8-dimensional !**

I.e., it is covariantly spanned by 8 types of quanta.

Its dimensions $r = 1, \dots, 8$ themselves, each, should be “1-dimensional”, *i.e., real-numbered*. However,

Principle #6:

This octuplication of dimensions is **repeatable!**

Why this?? Now, by the first octuplication, our N quanta q_n will be cast into 8 distinguishable **classes** according to their respective values of r : $q_n \equiv q_{r,\rho}$, where ρ is taking over the rest of individuality of our quantum $q_{r,\rho}$ beyond r . By repeating the same probability consideration which led us from the label n to the label *pair* r,ρ – this time, however, on the partial label ρ – we find

$$q_n \equiv q_{r,\rho} \equiv q_{r,s,\sigma} \equiv \dots$$

When bothering about the class labels r or s only, all further individualization, usually, will be kicked off: The quanta q_r , or $q_{r,s}$, etc., are **declared to be complete**: additional individuality at the always respecting level will experimentally not be observed – basta! (*Customary slogan in physics: "One electron is not distinguishable from another one."*)

By experiment, this special partition into classes of 8, each, yields **complete descriptions of nature**:

The 1st set $r = 1, \dots, 8$ yields

Quantum Gravity

The 2nd set $s = 1, \dots, 8$ adds the

Grand Unification

Let us treat both cases together. An 8-dimensional label r or s may be split into a triplet him and jkl , respectively, of 2-dimensional labels (for: $8 = 2^3$!). Our quantum, thus, will adopt the form

$$q_{r,s} \equiv q_{hijklm}$$

h = "Hermiticity spin"

i = ordinary spin

m = "particle / antiparticle spin"

QG

"internal"

j = "electromagnetic spin"

k = "strong spin"

l = "weak spin"

Provided **no further octuplication** is observed,
all $8^2 = 2^{3+3}$ dimensions will remain 1-dimensional,

i.e., r(eal)-numbered. *A need of using c-numbers, instead, would be a first hint towards a 3rd octuplication!*
In order to reproduce the **superposition principle of quantum theory** we need

Principle #7:

Only **linear** transformations are admitted.

*The most general linear transformation of such an r-numbered 8^2 -dimensional spinor can be expanded in terms of Kronecker products of Pauli's 4 matrices σ_μ (with the Lorentz-type label μ running from 0 to 3). However, σ_2 is imaginary! Hence, in our **expansion of degeneracies**, σ_2 has to be replaced by the r-numbered $i\sigma_2$. Let us use these Kronecker- σ 's as generators of some group. Then, we automatically obtain a **pseudo-unitary group**, namely a*

U(32, 32)

*for our set of $8^2 = 32+32$ types of quanta. Its split into 2 octets with labels r and s ($q_{r,s}$), then, is not necessarily a $U(4,4)^2$, a $U(4,4) \times U(8)$, et al., would do it as well. Until up to our 2nd level of **degeneracy expansion**, hence,*

All matter ("ordinary" as well as "dark") is composed of the
 8^2 entries of a fundamental "**periodic table**".

A priori, all our $8^2 = 64$ types of quanta of the 2nd degeneracy order, to some extent, will be more or less equivalent. But experimentalists are introducing differing **measuring units**. Therefore, let

Principle #8:

$U(32,32) \supset U(16,16)_{\text{covariant}} \oplus U(16,16)_{\text{contravariant}}$

This split of principle #8 comprises two aspects. One aspect is the split itself. Compare it with the split in Newtonian physics between 3-dimensional space on the one hand and time on the other hand. By the big factor $c = 3 \times 10^{10}$ cm/sec of the velocity of light, when c is set =1, $3 \text{ cm} = 10^{-10}$ sec only! By this big measuring factor, a Lorentz boost will become a much greater challenge to be executed by experiment than a simple rotation in isotropic space! Thus, for Newton, the dimensions of space and time still had been strictly separated. Attribute this split to some h_3 -component of some h -spin: $U(\cdot)_+ \oplus U(\cdot)_-$.

Then, its diagonal component h_3 , which will be identified to be proportional to **particle number**, will be easily measured. The non-diagonal boosts with h_1 and h_2 are identified [5] to be responsible for **Hermitean conjugation**. As they are inverting particle numbers, however, their experimental execution still is far outside our actual accessibility.

The other aspect of that split is that we assume both parts to transform **contravariantly** to each other with respect to some common subgroup $U(16,16)$ of both separate groups. This relative contravariance yields a positive count by the *linear* h -spin $U(1,1)$ Casimir $C_h^{(1)}$ for all quanta.

Such a $U(16,16)$, again, can be split further. An additive split could give 8 chiral groups containing our QG group (*right-hand side*):

$$U(16,16) \supset U(2,2)_1 \oplus \dots \oplus U(2,2)_8 \supset U(2,2)_J,$$

GUT

chiral "internal" force groups

QG

where the fixed label J is to distinguish our $U(2,2)_J$ of QG from the running label $a = 1$ to 8 of the **chiral** groups of the "internal" forces. The GUT contains generators **converting all forces into each other!**

Principle #9:

$$U(2,2)_J \supset U(1)_{\text{particle \#}} \times SU(2)_{\text{spin}} \times SU(1,1)_{a/b\text{-spin}}$$

with "a/b-spin" as shown in the definition of Dirac's spinors:

$$\psi = \begin{pmatrix} \mathbf{q}_{111} \\ \mathbf{q}_{121} \\ \mathbf{q}_{112} \\ \mathbf{q}_{122} \end{pmatrix} = \begin{pmatrix} \mathbf{a}_1^+ \\ \mathbf{a}_2^+ \\ \mathbf{b}_1^- \\ \mathbf{b}_2^- \end{pmatrix}$$

and

$$\psi^+ \equiv (\mathbf{q}_{211}, \mathbf{q}_{221}, \mathbf{q}_{212}, \mathbf{q}_{222}) \equiv (\mathbf{a}_1^-, \mathbf{a}_2^-, \mathbf{b}_1^+, \mathbf{b}_2^+),$$

Dirac's formalism is reproduced *in its "2nd-quantized" form* – with one crucial exception, however:

Dirac's spinors are carrying **external arguments**: $\psi(\underline{x}_\mu)$ or $\psi(\underline{p}_\mu)$, respectively.

Ours do *not* !!!

For, in QG, **every quantum q_{him} carries its own, fixed quantum numbers** ($= \pm 1/2$) [2] [3]. A diagonal set could be energy, the spin_3 component, and its (CMS-)space component Q_3 , e.g. In the *microcosmos* neither transversal spin_1 nor spin_2 nor their (CMS-)space components Q_1, Q_2 are measurable *simultaneously* with spin_3 and Q_3 .

Observe that the *additive* split of our original octet representation into the 2 Dirac spinors is separating **two** $U(2,2)$ subgroups from each other. One of them knows the covariant octet labels only and the other one merely the contravariant ones. This has implications:

- (1) Strictly speaking, representations are belonging to **2 different subrepresentation** $U(2,2)_1 \times U(2,2)_2 \supset U(2,2)_{\text{common}}$. With respect to their common $U(2,2)$ subgroup, they are not totally irreducible any more: **traces** are not kicked out!
- (2) And $C_h^{(1)}$, the **number of co- plus contravariant labels** of such a squared $U(2,2)$ representation **is fixed**, too!

Now, quantum field theory (QFT) is working with the inhomogeneous commutator type $[a^-, a^+] \propto \delta$. Hence its notion of a "**destruction** operator" for a^- and of a "**creation** operator" for a^+ . QFT, thus, is dropping $C_h^{(1)}$. By doing so, it is grossly violating one of the most sacrosanct principles of physics, its conservation principle:

Principle #10:

**Nothing falls from heaven,
Nothing gets lost.**

Not even quanta!

A “better” version would read $[a^-, a^+] \propto \delta \text{tr}(a^- a^+)$. The general **dropping of quantum singlets** in

QFT is a source of inconsistencies !

Its backward formulation $\delta \propto [a^-, a^+]$ is giving rise to those awful “**vacuum fluctuations**” trying to persuade us paradoxically that a vacuum is not empty, i.e., that matter is **created out of nothing** – simsalabim! Within a correct description according to our model *conserving the number of quanta*, however,

An **interaction** is nothing else than a **reshoveling** of elementary quanta

where, according to the rules of combinatorics, (also) **quantum singlets** are perpetually puffed out and reabsorbed. **QFT**, instead, **is the perpetualized quarrel with self-created inconsistencies**.

That **ignorance of quantum singlets by QFT** permits QFT indirectly to add ever more singlets to a representation when applying some non-compact “boost”-operator to it. Endlessly. Finally, a Lorentz-boosted particle might gain more energy than there is in the entire universe! For, QFT-representations are not confined, they are **infinite**. This, however, is contradicting human’s perception of nature.

In our model, a particle is a mere superposition effect of (*some giant number of*) interfering eigenstates of a **finite universe** [4]. For, if we accept our universe to be “irreducible” (*i.e., non-decomposable into smaller portions organized the same way*), then a particle, on principle, cannot split off without killing the entire universe! This is the reason why **parallel-world scenarios** in connection with the measuring process will remain pure science-fiction: The Copenhagen interpretation cannot be true ! Irreducibility, however, guarantees that our **constants of nature**, i.e., the eigenvalues of our Casimir operators, will be identical all over our universe. Otherwise, this would provide a hint that our universe is not “irreducible” itself but a member of some bigger agglomeration interacting with what we *imagined* to be “our” universe!

This, of course, does not prevent us from *approximating* a particle state by the same way we describe a universe. Briefly, every representation (*of particles, and also of universes if observable*) should remain finite.

When quantum singlets are taken into account, there is no need any more for infinite particle representation in order to conserve probability. The **positive-definite norm** absent in QFT, for defining a normalizable probability for its non-compact operations, here, is given by the fixed number $C_h^{(1)}$ of quanta within a (finite) representation. This, however, is no property of an $SU(2,2)$ subgroup; it needs at least the entire group $U(2,2)^2$ which is counting them. (Within the framework of QG, this count is performed by the difference(!) of the linear Casimirs of both $U(2,2)$ factors. In the $U(2,2)_{\text{common}}$, only their sum (i.e., the difference co- minus contravariant quanta) is available.) Sorry, it is a bit confusing.

But what is the reason for being allowed to measure the compact, diagonal h_3 component of Hermitecity-spin h – remember that h_3 is proportional to the particle number operator – but not to execute a transformation with its two non-compact components h_1 or h_2 ? The answer is:

“rigidity”!

As I told you already: Compare it with a Lorentz boost in relation to a rotation. While a rotation is working among equivalent spatial dimensions x,y,z , a Lorentz boost is mixing up, say, x with ct . It is that velocity-of-light factor $c = 3 \times 10^{10}$ cm/sec which, by c set =1, is shrinking 3cm to only 10^{-10} sec, thus making it much more cumbersome for an experimentalist to accelerate a system to some shifted Lorentz frame instead of easily rotating it within an isotropic space.

By considering *different* occupation numbers in the distribution of quanta,

Measuring units are defining differing degrees of “rigidity”.

It will be this “rigidity” – *but with some incomparably stronger, greater numerical factor than 10^{10}* – which is effectively preventing an experimentalist from “accelerating” Hermitecity by boosting it! The result of such a boost would be that particles could convert into antiparticles, e.g. As, in QG, $U(4,4)$ is our true group, such a conversion will happen within our universe [4] – however, it is a matter of probability.

Similar effects will be at work with the so called “internal” forces.

Principle #11:

“Internal” boosts are highly suppressed.

Like “Hermitecity boosts”.

But let us consider the a_{ijkl}^+ (ignoring ordinary spin s). Remember that the “standard model” needs 24 types of quarks and leptons, plus more than a dozen “fundamental” bosons. QG and its GUT extension are managing all that with just 8 types of comparable entities. And space-time is included – in the “SM” it is extra!

The application of the 3-components of h-, j-, k-, and l-spin on the a_{ijkl}^+ will yield **charge** values:

#	charge / spin	“up ₁ ” / “down ₁ ”		“up ₂ ” / “down ₂ ”		“leptonics”		“exotics”		
		a_{i211}^+	a_{i111}^+	a_{i222}^+	a_{i122}^+	a_{i212}^+	a_{i112}^+	a_{i221}^+	a_{i121}^+	
0.	$\mathbf{N} \equiv -\frac{2}{3} h_3$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	← particle number N
1.	$\mathbf{Q} \equiv +\frac{1}{3} h_3 - j_3$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	← electric charge Q
2.	$\mathbf{T} \equiv +\frac{1}{3} h_3 - k_3$	$-\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{2}{3}$	← strong charge T
3.	$\mathbf{L} \equiv \frac{1}{2} (k_3 - l_3)$	0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	← lepton number L

These are 4 forces resulting from a *multiplicative* split of our “internal” octet: Its 3 “internal” forces ($8 = 2^3$) are given by its “vector” part, while gravity is the “singlet” part (2^0) invariant with respect to “internal” activities. Our *octet* model ($8 = 4+4$!), however, still is predicting another 4 “internal” forces:

#	Kronecker spins									
4.	$\mathbf{\Lambda} \equiv \begin{pmatrix} j_3 \times k_3 - j_3 \times l_3 \\ +2j_3 - j_3 \times k_3 \times l_3 \end{pmatrix}$	0	0	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	← leptonic charge Λ
5.	$\mathbf{E} \equiv \frac{1}{2} \begin{pmatrix} j_3 \times k_3 - j_3 \times l_3 \\ -2j_3 + j_3 \times k_3 \times l_3 \end{pmatrix}$	0	0	0	0	0	0	$+\frac{1}{2}$	$-\frac{1}{2}$	← exotic charge E
6.	$\mathbf{A} \equiv \frac{1}{2} (k_3 \times l_3 - l_3)$	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	← strong charge A
7.	$\mathbf{M} \equiv \frac{1}{2} \begin{pmatrix} j_3 \times k_3 + j_3 \times l_3 \\ -2j_3 - j_3 \times k_3 \times l_3 \end{pmatrix}$	0	0	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	← strong charge M

The explicit derivation of these tables and their discussion, however, must be shifted to some specialized lecture on particles [5]. This includes the solution of the “quark confinement”.

This result shows up parallels to quarks. But our quanta **are no quarks**: Their energies are much too low! But in the universal “sea” of all our quanta, forces of all kind will be at work trying to saturate at least their “internal” bonds by predominantly pairing them to pairs, those quantum singlets QFT is systematically dropping. The only quantum numbers which *partly* might be left not equal to zero are those of QG:

Our **virgin universe** will behave like a **gas** whose “molecules” predominantly are pairs of the **4 types** $a_i^+ b_{i'}^+$, $a_i^+ a_{i'}^-$, $b_i^- b_{i'}^+$, $b_i^- a_{i'}^-$.

Those 16 pairs generating QG are **carrying units of energy +1, 0, 0, -1**, respectively.

We assumed that no single quanta will be left unpaired. Provided this happened in spite – *and if only because they did “not yet” find their partners* – then, those unpaired “**valence quanta**” among the “vapour molecules” of saturated “virgin matter”, obviously, are representing some kind of **condensation germs**, about which those building bricks of virgin matter are lumping together due to their Van-der-Waals-like forces – like water molecules of a cloud in sky around a particle of soot. In our case,

The droplets condensing out of that quantum steam “humidity” are (complete) **elementary particles**.

Those “bricks” $a_{ijkl}^+ b_{i'jkl}^+$, $a_{ijkl}^+ a_{i'jkl}^-$, $b_{ijkl}^- b_{i'jkl}^+$, $b_{ijkl}^- a_{i'jkl}^-$ of virgin matter are building up the **non-valence** parts of elementary particles.

The building bricks of virgin matter, as a matter of principle, are **not localizable**

because the space-time operators of QG are non-compact. For being measurable at least approximately, *due to the law of great numbers*, the “cooperation” of plenty of them would be necessary. On the other hand,

By their exactly measurable energies, at least the $a_i^+ b_{i'}^+$ and $b_i^- a_{i'}^-$ are executing **gravitation**.

Hence,

That diffuse “gas” made of the $a_i^+ b_i^+$, $a_i^+ a_i^-$, $b_i^- b_i^+$, $b_i^- a_i^-$
is representing (“Cold”)

Dark Matter.

All effects attributed to “vacuum polarisation” by QFT, hence, should be recalculated in terms of Dark Matter – **leaving the vacuum a vacuum!** And,

As particle masses are accumulating with their **non-valence parts**,

Higgs particles are not needed!

Briefly: **Physics** simply can be traced back to group theory, i.e., to **combinatorics + probability**.

- [1] C. Birkholz, Verhandl. DPG (VI) **48,1/AGPhil** 10.3 (Jena, 2013).
- [2] C. Birkholz, Verhandl. DPG (VI) **46,3/T** 25.1 (Karlsruhe, 2011). This is based on
C. Birkholz, “Weltbild ... im 3. Jahrtausend“, ISBN 978-3-00-030847-5 (2010).
Since 2010, new, additional relations have been uncovered allowing the old discoveries to be founded more stringently.
- [3] C. Birkholz, Verhandl. DPG (VI) **47,1/GR** 10.1 (Göttingen, 2012).
- [4] C. Birkholz, Verhandl. DPG (VI) **48,1/MP** 1.3 (Jena, 2013).
- [5] C. Birkholz, Verhandl. DPG (VI) **48,2/T** 25.3 (Dresden, 2013).

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